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DEPENDENT MIXED ACCEPTANCE SAMPLING PLANS AND THEIR EVALUATION

by

E. G. Schilling

and

H. F. Dodge

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1. INTRODUCTION

1.1 Introduction

Using the method developed in Technical Report No. N-26 [1] for determining certain needed joint probabilities, this report gives procedures for evaluating the operating characteristic curves and associated measures of dependent mixed acceptance sampling plans for the case of single specification limit, and known standard deviation, assuming a normal distribution. A useful generalized dependent plan is developed, using two attributes acceptance numbers rather than just one. Also included is a comparison of dependent mixed plans with other types of acceptance sampling plans.

1.2 The Mixed Plan

The choice between acceptance sampling by attributes and by variables has commonly been considered a first step in the application of sampling plans to specific problems in industry. The dichotomy is more apparent than real, however, since other alternatives exist in the combination of both attributes and variables results to determine the disposition of the lot. One such procedure is the so-called "mixed" variables-attributes acceptance plan. Mixed plans have been discussed by Bowker and Goode [2], Gregory and Resnikoff [3], and Savage [4],

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[1]This report is based in part on work being done in preparation of a doctoral thesis at Rutgers - The State University.

among others, and are provided for in MIL-STD-414 [5]. They are, in essence, double sampling procedures involving variables inspection of the first sample and subsequent attributes inspection if the variables inspection of the first sample does not lead to acceptance.

Mixed plans are of two types, so-called "independent" and "dependent" plans. Independent mixed plans maintain stochastic independence between the probabilities of the variables and attributes constituents of the procedure. Independent plans have conventionally been carried out as follows [2]:

1. Obtain first sample.
2. Test first sample against a given variables acceptance criterion and:
   a) Accept if the test meets the variables criterion.
   b) Resample if the test fails to meet the variables criterion.
3. Obtain a second sample if necessary (per 2(b)).
4. Test the second sample (only) against a given attributes criterion and accept or reject as indicated by the test.

Dependent mixed plans are those in which the probabilities of the variables and attributes constituents of the procedure are made dependent. The dependent procedure, as proposed by Savage [4], can be summarized as follows:

1. Obtain first sample.
2. Test first sample against a given variables acceptance criterion and:
   a) Accept if the test meets the variables criterion.
   b) If the test fails to meet the variables criterion:
      (1) Reject if the number of defectives in the first sample exceeds a given attributes criterion.
      (2) Otherwise resample.
3. Obtain a second sample if necessary (per 2(b)(2)).
4. Test the results for the first and second samples taken together against the given attributes criterion and accept or reject as indicated by the test.

Note that this procedure can be generalized by providing for the use of different attributes criteria in steps 2 and 4. Such a generalized dependent mixed plan is presented in Section 2.1 below.

The dependent plan provides the optimal procedure in terms of the size of average sample number (ASN) associated with the plan. Attention will be directed here to the evaluation of operating characteristic curves and associated measures of dependent mixed plans in the case of single specification limit, known standard deviation, when a normal distribution of product is assumed.

2. Advantages and Disadvantages of Mixed Plans

The assumption of normality inherent in most variables acceptance procedures has proved to be both their strength and their undoing. Perturbations in the production process or screening of defective product may make otherwise normally distributed product anything but normal. Whatever the source of non-normality, the possibility of submission of such product to standard variables plans is a serious consideration weighing against their use except under conditions where normality is well assured. Nonetheless, the reduction in sample size attendant with variables plans makes them particularly inviting.

The mixed variables-attributes plan achieves some of the reduction of sample size associated with a variables plan without some of the related disadvantages. The mixed procedure appeals to the psychology of inspectors by giving a questionable lot a second chance. In rejecting lots it is also often a decided psychological or legal advantage to be able to show actual defectives to the producer, a feature which can be had only by rejecting on an attributes basis. Truncated and non-normal distributions cannot be rejected for poor variables results alone, but only on the basis of defective units found in the attributes sample. Furthermore, with regard to acceptance-rejection decisions, the effect of changes in shape of distribution can be minimized by accepting only on variables evidence so good as to be practically beyond question for most distributions which might reasonably be presented to the plan. Thus, mixed plans provide a worthwhile alternative to variables plans used alone.

The principal advantage of a variables-attributes scheme over attributes alone is a reduction in sample size for the same protection. The variables aspect of the mixed plan also allows for a far more careful analysis of the distribution of product presented to the plan than would be possible with attributes inspection alone. Variables control charts kept on this information

from lot to lot can provide information on the variability and stability of product from lot to lot. Control charts should normally be used in conjunction with acceptance sampling procedures involving variables inspection.

With small first samples, the mixed plan provides an excellent form of surveillance inspection on product which is generally expected to be of good quality but which may, at times, show degradation. A small variables first sample can be employed to accept at relatively low values of percent defective and a second attributes sample then used to provide a definitive criterion for disposition of the lot if it is not accepted on the first sample.

Unfortunately, mixed plans do not provide the same protection against non-normality for acceptance as they do for rejection, since product is accepted at the first stage of the plan on a variables basis. It is possible, however, to minimize this disadvantage for product well within specification by designing the plan in such a way as to accept on a variables basis only product with distribution located far enough from the specification limit so that reasonable changes in the shape of the distribution will not cause appreciable changes in percent defective. In this way a tight variables criterion could be employed to minimize the effect of changes in shape of distribution on the operating characteristic curve of the plan.

In application, it is also conceivable that mixed plans might be more difficult to administer than either variables plans or attributes plans alone. As with all plans using variables criteria, a separate mixed plan must be developed for each characteristic to which it is applied. Any increase in complexity would, however, probably be compensated for by the advantages of the mixed procedure.

2. FORMULATION OF DEPENDENT MIXED PLANS

2.1 A Generalized Mixed Dependent Procedure

Given an upper specification limit[2], the inspection procedure for application of a single specification limit (U), known standard deviation (σ), dependent mixed plan is generalized here by allowing for two acceptance numbers. The first acceptance number (c_1) is applied to the attributes results of the first sample after acceptance by variables and before a second sample is taken. The second acceptance number (c_2) is applied to the combined first and second sample attributes results. As a special case, the two acceptance numbers may be made the same; this is the plan proposed by Savage [4]. Providing for the use of different acceptance numbers increases the flexibility and potential of the dependent mixed plan.

Let:

N = lot size

n_1 = first sample size

n_2 = second sample size

A = acceptance limit[3] on sample mean ($\bar{x}$)

c_1 = attributes acceptance number on first sample

c_2 = attributes acceptance number on second sample

Then the generalized plan would be carried out in the following manner:

1. Determine the parameters of the mixed plan: n_1, n_2, A, c_1, c_2.
2. Take a random sample of n_1 from the lot.

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[2]Symmetry obviates the necessity for parallel consideration of a lower specification limit.

[3]Of the several methods of specifying the variables constituent of known standard deviation (σ) variables plans, designation by sample size (n_1) and acceptance limit on the sample average (A) is used here since it simplifies the notation somewhat. Note that $A = (U - k\sigma)$ for upper specification limit (U) and standard variables acceptance factor k.

3. If the sample average $\bar{x} \leq A$, accept the lot.

4. If the sample average $\bar{x} > A$, examine the first sample for the number of defectives d_1 therein.

5. If $d_1 > c_1$, reject the lot.

6. If $d_1 \leq c_1$, take a second random sample of n_2 from the lot and determine the number of defectives d_2 therein.

7. If in the combined sample of $n = n_1 + n_2$, the total number of defectives $d = d_1 + d_2$ is such that $d \leq c_2$, accept the lot.

8. If $d > c_2$, reject the lot.

When semi-curtailed inspection[4] is employed, a desirable practice and normally to be recommended, the procedure remains the same, except that, if c_2 is exceeded at any time during the inspection of the second sample, inspection is stopped at once and the lot rejected.

2.2 Operating Characteristic (OC) Curves and Associated Measures

The four principal curves which describe the properties of an acceptance sampling plan for various percents defective are the operating characteristic or OC curve, the average sample number or ASN curve, the average total inspection or ATI curve, and the average outgoing quality or AOQ curve. The operation of mixed plans cannot be properly assessed until formulas for the ordinates of each of these curves, for given values of the true percent defective, are defined. In particular, attention will be directed here to Type B OC curves[5] (i.e. sampling from a process) since it is to this type OC curve that the values of joint probabilities evaluated in [1] apply. Let:

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[4] Semi-curtailed inspection involves stopping inspection of the second sample only upon rejection of the lot [6].

[5] Note that for Type B operating characteristic curves [8] and for associated measures, $P(i;n)$ should be determined using the binomial distribution.

P_a = probability of acceptance

ASN = average sample number

ASN_c = average sample number under semi-curtailed inspection[4]

ATI = average total inspection

AOQ = average outgoing quality (with replacement of all defectives),

and

P(V) = probability of V

$P_n(V,W)$ = probability of V and W in a sample of n

$P_n(V|W)$ = probability of V given W in a sample of n

P(i;n) = probability of i defectives in a sample of n.

Also, let:

μ = population (process) mean

σ = population (process) standard deviation (known)

p = population (process) fraction defective

$\bar{x}$ = sample mean.

Then, formulas[6] for the operating characteristic curve and associated measures of the general procedure are given in Table 1.

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[6]These formulas are developed by simple analogy with double sampling by attributes. In the case of ASN_c, Burr's formula for ASN_2 is employed; see I. W. Burr [7] p. 313.

TABLE 1

FORMULAS FOR MEASURES OF DEPENDENT MIXED PLANS*

Measure	Formula
P_a	$P_a = P(\bar{x} \le A) + \sum_{i=0}^{c_1} \sum_{j=0}^{c_2-i} P_{n_1}(i, \bar{x} > A)\, P(j; n_2)$
ASN	$ASN = n_1 + n_2 \sum_{i=0}^{c_1} P_{n_1}(i, \bar{x} > A)$
ASN_c	$ASN_c = n_1 + \sum_{i=0}^{c_1} P_{n_1}(i, \bar{x} > A) \Big[\frac{c_2-i+1}{p} \sum_{k=c_2-i+2}^{n_2+1} P(k; n_2+1) + n_2 \sum_{j=0}^{c_2-i} P(j; n_2) \Big]$
ATI	$ATI = ASN + (N-n_1) \sum_{i=c_1+1}^{n_1} P_{n_1}(i, \bar{x} > A) + (N-n_1-n_2)(1-P_a - \sum_{i=c_1+1}^{n_1} P_{n_1}(i, \bar{x} > A))$
AOQ	$AOQ = \frac{p}{N}\Big[P(\bar{x} \le A)(N-n_1) + (P_a - P(\bar{x} \le A))(N-n_1-n_2) \Big]$

* Except for ASN, all formulas are the same with or without curtailed inspection.

Since σ is assumed known, it is possible to evaluate the expressions shown in Table 1 using tables of $P_n(i,\bar{x} > A)$ for a "standard normal universe", i.e. $\mu = 0$, $\sigma = 1$. Such values are given in the appendix of this report for first sample size $n_1 = 5$. To accomplish this, the value of $P_n(i,\bar{x} > A)$ for a particular application can be found by transforming the variates involved to standard-normal-deviates by use of the familiar z transformation. This expresses the departure of given values from the population mean in units of the (known) standard deviation. Thus, the upper specification limit U is expressed as z_U where:

$$z_U = \frac{U-\mu}{\sigma}$$

and μ is the population mean of a normal distribution such that fraction defective p of the said distribution exceeds the upper specification limit, U. (See Figure 1, below.)

Thus:

$$P_n(i,\bar{x} > A) = P_n(i,\bar{z} > z_A)$$

where $\bar{z}$ and z_A are standard-normal-deviates such that:

$$\bar{z} = \frac{\bar{x}-\mu}{\sigma}$$

and

$$z_A = \frac{A-\mu}{\sigma} .$$

The tables in the appendix are entered with these values for the mean and the acceptance limit.

The values shown in the appendix were calculated using the method indicated in Technical Report No. N-26 [1]. Similar tables for sample sizes 4 to 10 are presented in Technical Report No. N-28 [9], which also discusses

the accuracy of the tabulated values. For sample size 5, the values shown in the appendix are believed to be accurate to at least four places when $c = 0$ and to at least three, and perhaps four, places when $c = 1$ and $c = 2$.

3. COMBINING VARIABLES AND ATTRIBUTES PLANS

3.1 General Considerations

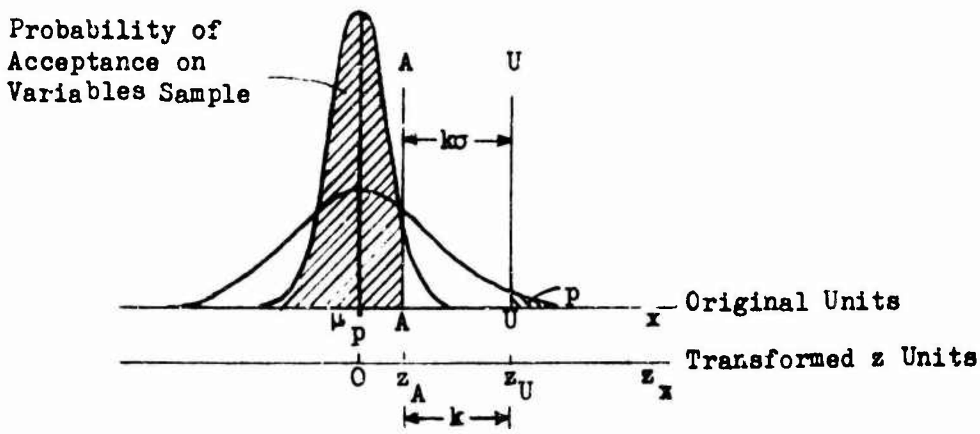
Variables plans for single upper specification limit (U) and known standard deviation (σ) are usually specified in one of the following three ways:

1. An acceptance limit (A) is specified for a given sample size (n) and lots are accepted if the mean of a sample of n does not exceed the limit; otherwise they are rejected. This method is often used in practice since it minimizes the computations involved on the part of the inspector.
2. A value of k is given for a particular sample size n. Lots are accepted if for the mean ($\bar{x}$) of a sample of n, $\frac{(U-\bar{x})}{\sigma} \geq k$; otherwise they are rejected. This is Form 1 of MIL-STD-414 [5].
3. Values of M are given for a particular sample size n. For a given sample mean of n observations the statistic
$$Q_U = \frac{(U-\bar{x})v}{\sigma}, \text{ where } v = \sqrt{\frac{n}{n-1}},$$
is calculated and an estimated percent defective P_U obtained from a table for the value obtained for Q_U. Lots are accepted if $P_U \leq M$, otherwise they are rejected. This is Form 2 of MIL-STD-414.

Figure 1 shows the relationship of k and A for a given distribution of product with mean μ_p associated with fraction defective p. It also displays the role of the transformed variables z_A and z_U.

FIGURE 1

RELATIONSHIP OF k AND A



The following discussion will be in terms of the first of the three methods mentioned above since this simplifies the notation somewhat. Variables plans specified in terms of the second or third methods can be converted to the first method using:

$A = U - k\sigma$,

or

$$A = U - \sqrt{\frac{n-1}{n}}\,K\sigma,\ K \text{ such that } \int_K^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt = \frac{M}{100} \text{ in the notation of MIL-}$$

STD-414 [5], respectively. For purposes of this report, it is assumed that in application, variables plans will be converted to the first method.

In combining any two variables and attributes plans in a dependent mixed plan, the formulas of Table 1 define the probability of acceptance, or OC curve, and associated measures of the combined plan. The formulas simplify greatly when $c_1 = 0$. Procedures for combining variables and attributes plans will be illustrated for cases in which $c_1 = c_2$ and also for $c_1 \neq c_2$. Extension to other combinations of acceptance numbers is straightforward.

3.2 Example 1, Specified Plan, $c_1 = c_2$

To illustrate the method of evaluating plans when $c_1 = c_2$ suppose, arbitrarily, that the OC curve of the following specified dependent plan is desired:

$n_1 = 5$, $k = 2$

$n_2 = 20$, $c_1 = c_2 = 0$

1. Since $c_1 = 0$, the formula for probability of acceptance simplifies to:
$$P_a = P(\bar{x} \le A) + P_5(0,\bar{x} > A)\ P(0;20)$$
2. The probabilities on the right hand side are determined as follows:
   (a) $P(i;n)$ by direct calculation from the binomial distribution or from tables of that distribution,
   (b) $P_n(i;\bar{x} > A)$ must be converted to $P_n(i,\bar{z} > z_A)$, as indicated above, to make use of the tables in the appendix. Note that z_A is expressed in units of the known population standard deviation.
   (c) $P(\bar{x} \le A)$ is determined from the usual tables of the standard normal distribution. In looking up values of this probability it is necessary to adjust for the fact that the standard deviation of the distribution of sample means is:
$$\sigma_{\bar{x}} = \sigma/\sqrt{n}\ .$$
   Thus, in terms of standard values:
$$P(\bar{x} \le A) = P(\bar{z} \le \sqrt{n_1} z_A)$$
   since z_A is expressed in terms of the known population standard deviation, σ.

   The formula, expressed in standard units then becomes:
$$P_a = P(\bar{z} \le \sqrt{n_1} z_A) + P_5(0,\bar{z} > z_A) P(i;n)$$
3. Computation is, then, as shown in Table 2. Each row of the table represents a given fraction defective p associated with a corresponding population mean μ_p. As shown in Figure 1, the values of z_U and z_A are standard normal deviates from μ_p. Note that for plans specified in terms of k, z_A can be determined as
$$z_A = z_U - k.$$
   The remaining columns follow from the formula for probability of acceptance.

3.3 Example 2, Specified Plan, $c_1 \neq c_2$

As an illustration of the method of evaluating plans with $c_1 \neq c_2$ suppose, arbitrarily, that the OC curve of the following specified dependent plan is desired:

$n_1 = 5$ $\quad$ $k = 2$

$n_2 = 20$ $\quad$ $c_1 = 1$, $c_2 = 2$

1. The formula is:

$$P_a = P(\bar{x} \le A) + \sum_{i=0}^{1} \sum_{j=0}^{2-i} P_5(i,\bar{x} > A)\, P(j;20)$$

2. Computation is then as shown in Table 3, set up in the same manner as Example 1 above.

3.4 <u>Example 3. Combining Published Plans</u>

To illustrate the potential of the method for determining the OC curve of a combination of any two plans, suppose the following two plans are combined after the manner of MIL-STD-414:

MIL-STD-414 [5] Code F (AQL = 4.0): $n = 5$, $k = 1.20$

MIL-STD-105D [10] Code F (AQL = 4.0 tightened): $n = 20$, $c = 1$

Note that in combining published plans in the manner of MIL-STD-414, $n_2 = 15$ in the calculations since 5 units are contributed by the first sample to the attributes determination.

Let $c_1 = c_2 = 1$.

The combined Type B OC curve could be derived as follows:

1. The formula is:

$$P_a = P(\bar{x} \le A) + \sum_{i=0}^{1} \sum_{j=0}^{1-i} P_5(i,\bar{x} > A)\, P(j;15)$$

2. Computation is then as shown in Table 4, set up in the same manner as the examples above.

These examples illustrate the relatively simple calculation of the OC curve of a dependent mixed plan if tables of $P_n(i,\bar{x} > A)$ are available.

TABLE 2

EXAMPLE 1

CALCULATION OF OC CURVE FOR COMBINED VARIABLES AND ATTRIBUTES PLAN, $c_1 = c_2 = 0$

$n_1 = 5$, $k = 2$

$n_2 = 20$, $c_1 = c_2 = 0$

A	B	C	D	E	F	G	P_a
p	z_U	z_A	$P(\bar{z} \le \sqrt{n_1} z_A)$	$P_5(0, \bar{z} > z_A)$	$P(0;20)$	ExF	D+G
.005	2.58	.58	.9032	.0851	.9048	.0770	.9802
.008	2.41	.41	.8204	.1551	.8516	.1321	.9525
.01	2.33	.33	.7704	.1967	.8179	.1609	.9313
.02	2.05	.05	.5438	.3736	.6676	.2494	.7932
.05	1.64	-.36	.2090	.5705	.3585	.2045	.4135
.10	1.28	-.72	.0537	.5389	.1216	.0655	.1192
.107	1.24	-.76	.0446	.5271	.1045	.0551	.0997
.15	1.04	-.96	.0158	.4285	.0388	.0166	.0324
.20	0.84	-1.16	.0047	.3231	.0115	.0037	.0084

TABLE 3

EXAMPLE 2

CALCULATION OF OC CURVE FOR COMBINED VARIABLES AND

ATTRIBUTES PLAN, $c_1 = 1$, $c_2 = 2$

$n_1 = 5$, $k = 2$

$n_2 = 20$, $c_1 = 1$, $c_2 = 2$

A	B	C	D	E	F	G	H	I	P_a
p	z_U	z_A	$P(\overline{z} \le \sqrt{n_1} z_A)$	$P_5(0, \overline{z} > A)$	$\sum_{j=0}^{2} P(j;20)$	$P_5(1, \overline{z} > A)$	$\sum_{j=0}^{1} P(j;20)$	EF+GH	D+I
.005	2.58	.58	.9032	.0851	.9998	.0120	.9953	.0970	1.0000
.01	2.33	.33	.7704	.1967	.9990	.0326	.9831	.2286	.9990
.02	2.05	.05	.5438	.3736	.9929	.0780	.9401	.4443	.9881
.05	1.64	-.36	.2090	.5705	.9245	.1965	.7358	.6720	.8810
.10	1.28	-.72	.0537	.5389	.9769	.3259	.3917	.4924	.5461
.15	1.04	-.96	.0158	.4285	.4049	.3908	.1756	.2421	.2579
.20	.84	-1.16	.0047	.3231	.2061	.4094	.0692	.0949	.0996

TABLE 4

EXAMPLE 3

CALCULATION OF OC CURVE FOR COMBINED VARIABLES AND

ATTRIBUTES PLAN, $c_1 = c_2 = 1$

$n_1 = 5$, $k = 1.20$

$n_2 = 20$, $c_1 = c_2 = 1$

A	B	C	D	E	F	G	H	I	P_a
p	z_U	z_A	$P(\bar{z} \le \sqrt{n_1} z_A)$	$P_5(0, \bar{z} > z_A)$	$\sum_{j=0}^{1} P(j;15)$	$P_5(1, \bar{z} > z_A)$	$P(0;15)$	EF+GH	D+I
.005	2.58	1.38	.9990	.0005	.9974	.0005	.9278	.0010	1.000
.01	2.33	1.13	.9943	.0027	.9904	.0027	.8601	.0050	.999
.02	2.05	0.85	.9713	.0128	.9647	.0135	.7386	.0223	.994
.05	1.64	0.44	.8389	.0647	.8290	.0789	.4633	.0902	.929
.10	1.28	0.08	.5714	.1420	.5490	.2097	.2059	.1211	.692
.15	1.04	-0.16	.3594	.1736	.3186	.3047	.0874	.0819	.441
.20	0.84	-0.36	.2090	.1741	.1671	.3550	.0352	.0416	.251

4. COMPARISON OF PLANS

4.1 Comparison of Independent and Dependent Mixed Plans

Suppose a comparison is made between an independent and a dependent plan which have "essentially" the same OC curve. A criterion for comparison then becomes the average sample number of the two plans. The probability of acceptance and average sample number of an independent mixed plan can be calculated [11] as:

$$P_a = P_{n_1}(\bar{x} \le A) + P_{n_1}(\bar{x} > A) \sum_{j=0}^{c_2} P(j;n_2)$$

$$ASN = n_1 + n_2 P_{n_1}(\bar{x} > A)$$

Now, if the two plans have the same first stage variables plan and attributes acceptance number c_2 (where for the dependent plan $c_1 \le c_2$), the seoond sample size of the independent plan will be greater than that of the dependent plan since:

$$P_a \text{ (independent)} = P_a \text{ (dependent)}$$

$$P(\bar{x} \le A) + P(\bar{x} > A) \sum_{j=0}^{c_2} P(j;n_2) = P(\bar{x} \le A) + \sum_{i=0}^{c_1} \sum_{j=0}^{c_2-i} P_{n_1}(i, \bar{x} > A)\, P(j;n_2')$$

$$P(\bar{x} > A) \sum_{j=0}^{c_2} P(j;n_2) = P(\bar{x} > A) \sum_{i=0}^{c_1} \sum_{j=0}^{c_2-i} P_{n_1}(i|\bar{x} > A)\, P(j;n_2')$$

$$\sum_{j=0}^{c_2} P(j;n_2) = \sum_{j=0}^{c_2-c_1} \left[\sum_{i=0}^{c_1} P_{n_1}(i|\bar{x} > A)\right] P(j;n_2') + \sum_{j=c_2-c_1+1}^{c_2} \left[\sum_{i=0}^{c_2-j} P_{n_1}(i|\bar{x} > A)\right] P(j;n_2')$$

But,

$$\sum_{i=0}^{c_1} P_{n_1}(i|\bar{x} > A) \leq 1$$

and

$$\sum_{i=0}^{c_2-j} P_{n_1}(i|\bar{x} > A) \leq 1 \; .$$

So to maintain the equality

$$\sum_{j=0}^{c_2} (j;n_2) \leq \sum_{j=0}^{c_2} (j;n_2')$$

which can only be achieved if $n_2 \geq n_2'$ for a given $p < .5$.

Therefore, for the same probability of acceptance, i.e. the same OC curve, the independent plan requires a larger second sample size. But even if the second sample size of the dependent plan is kept the same as that of the independent plan, the ASN of the dependent plan will be lower since:

ASN (independent) ≥ ASN (dependent),

$$n_1 + P(\bar{x} > A)n_2 \geq n_1 + n_2 \sum_{i=0}^{c_1} P_{n_1}(i,\bar{x} > A),$$

$$P(\bar{x} > A) \geq \sum_{i=0}^{c_1} P_{n_1}(i,\bar{x} > A).$$

Thus, the dependent plan is superior to the independent plan in terms of the same protection with a smaller sample size.

The difference in average sample number can become quite large if particularly bad quality is submitted to the plan and if, as seems customary, the independent plan has no provision for rejection on an attributes basis immediately after taking the first sample and before taking the second sample. Thus, in the event of

poor quality the attributes plan is utilized to a greater extent in the independent scheme than in the dependent procedure with further possible increase in the average sample number.

As an example of the superiority of dependent plans, consider the first of the examples given above:

$$n_1 = 5,\ k = 2;\ n_2 = 20,\ c_1 = c_2 = 0.$$

The probability of acceptance and average sample numbers were calculated for the specified mixed plan, assuming it to be carried out in dependent and independent form. A comparison of the results for the dependent and independent procedures is shown in Table 5.

TABLE 5

COMPARISON OF P_a AND ASN FOR A SPECIFIED MIXED PLAN

APPLIED IN DEPENDENT AND INDEPENDENT FORM

$n_1 = 5,\ n_2 = 20$

$k = 2,\ c_1 = c_2 = 0$

p	Dependent P_a	Dependent ASN	Independent P_a	Independent ASN
.005	.980	6.7	.991	6.9
.01	.931	8.9	.958	9.6
.02	.793	12.5	.848	14.1
.05	.414	16.4	.493	20.8
.10	.119	15.8	.169	23.9
.15	.032	13.6	.054	24.7
.20	.008	11.5	.016	24.9

4.2 Comparison of Mixed With Other Type Plans

As an indication of the relative merit of mixed plans, variables plans, and single and double sampling attributes plans were matched as closely as possible to the same dependent mixed plan:

$n_1 = 5$ $\quad$ $k = 2$

$n_2 = 20$ $\quad$ $c = 0$,

which was discussed above. These plans were matched as closely as possible at the two points:

$p_1 = .008$ $\quad$ $P_a = .953$

$p_2 = .107$ $\quad$ $P_a = .098$

which lie on the OC curve of the mixed plan. Due to inherent differences in the shape of the various OC curves, exact matches could not be obtained; however, all the plans obtained show probability of acceptance within ± 0.015 of the mixed plan at these points. The results are shown in Table 6.

TABLE 6

COMPARISON OF VARIOUS PLANS TO MATCH
$p_1 = .008$ $\quad$ $P_a = .953$
$p_2 = .107$ $\quad$ $P_a = .098$

Plan	Criteria	Prob. of Acceptance p=.008	p=.107	Avg. Sample No. p=.008	p=.107
Dependent Mixed	n_1=5, k=2 n_2=20, c=0	.953	.098	8.1	15.5
Variables	n=6, k=1.75	.947	.106	6	6
Attributes (Single)	n=37, c=1	.964	.084	37	37
Attributes (Double)	n_1=21, c_1=0 n_2=42, c_2=1	.947	.096	26.9	30.8

Comparison of the average sample number at these points for the various plans gives a rough indication of the superiority of mixed plans over either single or double sampling attributes plans. Also, it would appear that for low percents defective the average sample number for the mixed plan approaches that of the variables plan as illustrated in the following tabulation:

	Probability of Acceptance			Avg. Sample Number		
	$p = \sim 0$	$p=.005$	$p=.01$	$p= \sim 0$	$p=.005$	$p=.01$
Dependent Mixed	1.0	.980	.931	5.0	6.7	8.9
Variables	1.0	.975	.922	6	6	6
Single Attributes	1.0	.984	.947	37	37	37
Double Attributes	1.0	.981	.922	21	24.9	28.2

This is reasonable, since if "perfect" product (within the constraint of the assumption of normality) were submitted to both plans it would be accepted on the first stage of the mixed procedure resulting in an average sample number of 5 compared to the variables ASN of 6.

5. CONCLUSION

This paper presents a method for evaluating the operating characteristic curves and associated measures of dependent mixed acceptance sampling plans for the case of single specification limit, and standard deviation known, assuming an underlying normal distribution. Tables of joint probabilities $P_n(i,\bar{x} > A)$ necessary for evaluation of the properties of such plans with small first sample sizes are given in Technical Report No. N-28 [9] for sample sizes 4 (1) 10. These tables were computed by a method indicated in Technical Report No. N-26 [1]. Thus, the present report, together with the two companion reports, provides the basis for the implementation of this important, but as yet not effectively utilized, class of sampling plans.

Mixed plans maintain some of the most desirable features of variables and also of attributes sampling procedures without many of the related disadvantages. The assumption of a normal distribution is less restrictive fo mixed than for variables plans. At the same time average sample numbers are much lower than for attributes plans affording the same protection. Thus, these plans are more than a mixture - rather, an alloyage, a fusion of the constituents into a new, in some sense stronger whole.

6. REFERENCES

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APPENDIX - TABLES

JOINT PROBABILITY OF SAMPLE MEAN GREATER THAN z_A AND EXACTLY i DEFECTIVES IN SAMPLES FROM A NORMAL DISTRIBUTION (μ=0,σ=1)

(z_A-DEVIATION OF ACCEPTANCE LIMIT, A, FROM PROCESS MEAN IN UNITS OF KNOWN STANDARD DEVIATION OF INDIVIDUALS)

$n = 5$
$i = 0$

z_A	FRACTION DEFECTIVE, p .005	.01	.02	.05	.10	.15	.20
-2.50	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.45	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.40	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.35	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.30	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.25	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.20	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.15	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.10	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.05	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-2.00	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-1.95	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-1.90	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-1.85	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-1.80	.9752	.9510	.9039	.7738	.5905	.4437	.3277
-1.75	.9752	.9509	.9039	.7737	.5904	.4437	.3276
-1.70	.9752	.9509	.9038	.7737	.5904	.4436	.3276
-1.65	.9751	.9509	.9038	.7737	.5904	.4436	.3276
-1.60	.9751	.9508	.9037	.7737	.5903	.4435	.3275
-1.55	.9750	.9507	.9037	.7735	.5902	.4434	.3274
-1.50	.9749	.9506	.9035	.7734	.5901	.4433	.3273
-1.45	.9747	.9504	.9033	.7732	.5899	.4431	.3271
-1.40	.9744	.9501	.9030	.7729	.5896	.4428	.3268
-1.35	.9740	.9497	.9027	.7725	.5892	.4425	.3264
-1.30	.9734	.9492	.9021	.7720	.5887	.4419	.3259
-1.25	.9727	.9484	.9013	.7712	.5879	.4412	.3252
-1.20	.9716	.9473	.9003	.7701	.5869	.4401	.3242
-1.15	.9702	.9459	.8989	.7687	.5855	.4388	.3228
-1.10	.9683	.9440	.8970	.7669	.5836	.4370	.3211
-1.05	.9658	.9416	.8945	.7644	.5812	.4346	.3188
-1.00	.9626	.9383	.8913	.7612	.5780	.4315	.3159
-0.95	.9584	.9342	.8871	.7571	.5740	.4276	.3121
-0.90	.9532	.9289	.8819	.7518	.5689	.4227	.3075
-0.85	.9466	.9223	.8753	.7453	.5626	.4167	.3018
-0.80	.9384	.9142	.8672	.7373	.5548	.4093	.2949
-0.75	.9285	.9043	.8573	.7275	.5454	.4004	.2867
-0.70	.9165	.8923	.8453	.7158	.5342	.3899	.2771
-0.65	.9022	.8780	.8311	.7018	.5209	.3776	.2660
-0.60	.8854	.8613	.8144	.6855	.5055	.3634	.2533
-0.55	.8659	.8418	.7951	.6666	.4878	.3473	.2391
-0.50	.8436	.8195	.7729	.6451	.4678	.3294	.2235

FRACTION DEFECTIVE, p

z_A	.005	.01	.02	.05	.10	.15	.20
-0.45	.8182	.7942	.7478	.6208	.4456	.3096	.2067
-0.40	.7899	.7660	.7198	.5939	.4211	.2883	.1889
-0.35	.7586	.7348	.6890	.5644	.3947	.2656	.1703
-0.30	.7245	.7008	.6554	.5325	.3666	.2419	.1513
-0.25	.6877	.6642	.6193	.4985	.3371	.2176	.1323
-0.20	.6486	.6254	.5811	.4628	.3067	.1930	.1137
-0.15	.6075	.5846	.5411	.4258	.2758	.1687	.0959
-0.10	.5649	.5424	.4998	.3880	.2449	.1452	.0792
-0.05	.5213	.4992	.4578	.3500	.2146	.1227	.0639
0.00	.4771	.4557	.4155	.3123	.1854	.1018	.0503
0.05	.4331	.4123	.3736	.2755	.1577	.0828	.0386
0.10	.3897	.3696	.3326	.2401	.1320	.0659	.0287
0.15	.3475	.3282	.2930	.2066	.1086	.0511	.0206
0.20	.3070	.2886	.2554	.1754	.0876	.0387	.0143
0.25	.2685	.2512	.2201	.1468	.0694	.0285	.0095
0.30	.2326	.2163	.1875	.1210	.0537	.0203	.0060
0.35	.1993	.1842	.1577	.0982	.0407	.0140	.0036
0.40	.1690	.1551	.1311	.0784	.0300	.0093	.0020
0.45	.1418	.1291	.1075	.0615	.0216	.0059	.0010
0.50	.1176	.1061	.0869	.0474	.0151	.0036	.0005
0.55	.0964	.0861	.0693	.0358	.0102	.0020	.0002
0.60	.0781	.0691	.0545	.0265	.0066	.0011	.0001
0.65	.0625	.0546	.0421	.0191	.0042	.0005	.0000
0.70	.0494	.0426	.0321	.0135	.0025	.0002	.0000
0.75	.0386	.0328	.0240	.0093	.0014	.0001	.0000
0.80	.0297	.0249	.0177	.0063	.0008	.0000	.0000
0.85	.0226	.0186	.0128	.0041	.0004	.0000	.0000
0.90	.0169	.0136	.0091	.0026	.0002	.0000	.0000
0.95	.0125	.0099	.0063	.0016	.0001	.0000	.0000
1.00	.0091	.0070	.0043	.0009	.0000	.0000	.0000
1.05	.0066	.0049	.0028	.0005	.0000	.0000	.0000
1.10	.0046	.0034	.0018	.0003	.0000	.0000	.0000
1.15	.0032	.0023	.0012	.0001	.0000	.0000	.0000
1.20	.0022	.0015	.0007	.0001	.0000	.0000	.0000
1.25	.0015	.0010	.0004	.0000	.0000	.0000	.0000
1.30	.0010	.0006	.0003	.0000	.0000	.0000	.0000
1.35	.0006	.0004	.0001	.0000	.0000	.0000	.0000
1.40	.0004	.0002	.0001	.0000	.0000	.0000	.0000
1.45	.0003	.0001	.0000	.0000	.0000	.0000	.0000
1.50	.0002	.0001	.0000	.0000	.0000	.0000	.0000
1.55	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.60	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.65	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.75	.0000	.0000	.0000	.0000	.0000	.0000	.0000

JOINT PROBABILITY OF
SAMPLE MEAN GREATER THAN z_A AND EXACTLY i DEFECTIVES
IN SAMPLES FROM A NORMAL DISTRIBUTION ($\mu=0, \sigma=1$)

(z_A-DEVIATION OF ACCEPTANCE LIMIT, A, FROM PROCESS MEAN
IN UNITS OF KNOWN STANDARD DEVIATION OF INDIVIDUALS)

n = 5
i = 0

z_A	FRACTION DEFECTIVE, p .005	.01	.02	.05	.10	.15	.20
-2.50	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.45	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.40	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.35	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.30	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.25	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.20	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.15	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.10	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.05	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-2.00	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.95	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.90	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.85	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.80	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.75	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.70	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.65	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.60	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.55	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.50	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.45	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.40	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.35	.0245	.0480	.0922	.2036	.3280	.3915	.4096
-1.30	.0245	.0480	.0922	.2036	.3280	.3915	.4095
-1.25	.0245	.0480	.0922	.2036	.3280	.3914	.4095
-1.20	.0245	.0480	.0922	.2036	.3280	.3914	.4094
-1.15	.0245	.0480	.0922	.2036	.3280	.3914	.4094
-1.10	.0245	.0480	.0922	.2036	.3279	.3913	.4092
-1.05	.0245	.0480	.0922	.2035	.3279	.3912	.4090
-1.00	.0245	.0480	.0922	.2035	.3278	.3910	.4087
-.95	.0245	.0480	.0922	.2035	.3277	.3907	.4083
-.90	.0245	.0480	.0922	.2034	.3275	.3904	.4077
-.85	.0245	.0480	.0921	.2034	.3272	.3898	.4068
-.80	.0244	.0480	.0921	.2032	.3269	.3891	.4055
-.75	.0244	.0479	.0921	.2031	.3263	.3880	.4038
-.70	.0244	.0479	.0920	.2028	.3256	.3866	.4015
-.65	.0244	.0479	.0919	.2025	.3246	.3846	.3984
-.60	.0244	.0479	.0918	.2020	.3232	.3820	.3944
-.55	.0244	.0478	.0917	.2014	.3213	.3786	.3892
-.50	.0244	.0477	.0914	.2005	.3189	.3743	.3827

FRACTION DEFECTIVE, p

z_A	.005	.01	.02	.05	.10	.15	.20
-.45	.0243	.0476	.0911	.1994	.3158	.3688	.3745
-.40	.0243	.0475	.0907	.1979	.3119	.3619	.3646
-.35	.0242	.0473	.0902	.1961	.3070	.3535	.3526
-.30	.0241	.0470	.0896	.1937	.3010	.3434	.3385
-.25	.0240	.0467	.0887	.1908	.2937	.3314	.3222
-.20	.0238	.0462	.0876	.1872	.2851	.3175	.3038
-.15	.0236	.0457	.0863	.1830	.2750	.3016	.2832
-.10	.0233	.0450	.0847	.1779	.2634	.2839	.2608
-.05	.0230	.0443	.0828	.1720	.2503	.2643	.2369
0.00	.0226	.0433	.0806	.1653	.2358	.2433	.2120
.05	.0221	.0422	.0780	.1578	.2199	.2210	.1865
.10	.0215	.0409	.0751	.1494	.2029	.1980	.1612
.15	.0208	.0394	.0718	.1402	.1850	.1746	.1365
.20	.0201	.0377	.0681	.1304	.1666	.1514	.1130
.25	.0193	.0359	.0642	.1201	.1479	.1289	.0914
.30	.0183	.0339	.0599	.1093	.1293	.1076	.0720
.35	.0173	.0317	.0555	.0984	.1113	.0879	.0552
.40	.0162	.0295	.0508	.0875	.0941	.0701	.0410
.45	.0151	.0271	.0461	.0767	.0781	.0546	.0295
.50	.0139	.0247	.0414	.0664	.0636	.0413	.0204
.55	.0127	.0223	.0367	.0565	.0507	.0304	.0136
.60	.0115	.0199	.0321	.0474	.0394	.0216	.0087
.65	.0102	.0175	.0278	.0391	.0299	.0149	.0054
.70	.0091	.0153	.0237	.0317	.0221	.0099	.0031
.75	.0079	.0132	.0200	.0252	.0159	.0063	.0018
.80	.0068	.0112	.0166	.0196	.0111	.0038	.0009
.85	.0058	.0094	.0135	.0150	.0075	.0022	.0005
.90	.0049	.0078	.0109	.0112	.0049	.0013	.0002
.95	.0041	.0063	.0086	.0081	.0031	.0007	.0001
1.00	.0034	.0051	.0067	.0058	.0018	.0003	.0001
1.05	.0027	.0040	.0051	.0040	.0011	.0002	.0000
1.10	.0022	.0031	.0038	.0027	.0006	.0001	.0000
1.15	.0017	.0024	.0028	.0017	.0003	.0000	.0000
1.20	.0013	.0018	.0020	.0011	.0002	.0000	.0000
1.25	.0010	.0013	.0014	.0007	.0001	.0000	.0000
1.30	.0008	.0010	.0010	.0004	.0000	.0000	.0000
1.35	.0006	.0007	.0006	.0002	.0000	.0000	.0000
1.40	.0004	.0005	.0004	.0001	.0000	.0000	.0000
1.45	.0003	.0003	.0003	.0001	.0000	.0000	.0000
1.50	.0002	.0002	.0002	.0000	.0000	.0000	.0000
1.55	.0001	.0001	.0001	.0000	.0000	.0000	.0000
1.60	.0001	.0001	.0001	.0000	.0000	.0000	.0000
1.65	.0001	.0001	.0000	.0000	.0000	.0000	.0000
1.70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.75	.0000	.0000	.0000	.0000	.0000	.0000	.0000

JOINT PROBABILITY OF
SAMPLE MEAN GREATER THAN z_A AND EXACTLY i DEFECTIVES
IN SAMPLES FROM A NORMAL DISTRIBUTION (μ=0,σ=1)

(z_A-DEVIATION OF ACCEPTANCE LIMIT, A, FROM PROCESS MEAN
IN UNITS OF KNOWN STANDARD DEVIATION OF INDIVIDUALS)

n=5
i=0

	FRACTION DEFECTIVE, p						
z_A	.005	.01	.02	.05	.10	.15	.20
-2.50	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.45	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.40	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.35	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.30	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.25	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.20	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.15	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.10	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.05	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-2.00	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.95	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.90	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.85	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.80	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.75	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.70	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.65	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.60	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.55	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.50	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.45	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.40	.0002	.0010	.0038	.0214	.0729	.1382	.2048
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-1.25	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.20	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.15	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.10	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.05	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-1.00	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.95	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.90	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.85	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.80	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.75	.0002	.0010	.0038	.0214	.0729	.1382	.2048
-.70	.0002	.0010	.0038	.0214	.0729	.1382	.2047
-.65	.0002	.0010	.0038	.0214	.0729	.1382	.2046
-.60	.0002	.0010	.0038	.0214	.0729	.1381	.2045
-.55	.0002	.0010	.0038	.0214	.0729	.1380	.2043
-.50	.0002	.0010	.0038	.0214	.0729	.1380	.2041

z_A	.005	.01	.02	.05	.10	.15	.20
	FRACTION DEFECTIVE, p						
-.45	.0002	.0010	.0038	.0214	.0728	.1378	.2037
-.40	.0002	.0010	.0038	.0214	.0728	.1376	.2031
-.35	.0002	.0010	.0038	.0214	.0727	.1373	.2022
-.30	.0002	.0010	.0038	.0214	.0726	.1369	.2010
-.25	.0002	.0010	.0038	.0214	.0725	.1363	.1994
-.20	.0002	.0010	.0038	.0214	.0723	.1355	.1972
-.15	.0002	.0010	.0038	.0214	.0720	.1343	.1943
-.10	.0002	.0010	.0038	.0213	.0716	.1328	.1906
-.05	.0002	.0010	.0038	.0212	.0710	.1308	.1859
0.00	.0002	.0010	.0038	.0212	.0703	.1283	.1800
.05	.0002	.0010	.0037	.0210	.0693	.1251	.1729
.10	.0002	.0010	.0037	.0209	.0681	.1212	.1645
.15	.0002	.0010	.0037	.0207	.0665	.1165	.1549
.20	.0002	.0010	.0037	.0204	.0646	.1109	.1439
.25	.0002	.0010	.0037	.0201	.0623	.1046	.1318
.30	.0002	.0010	.0036	.0196	.0596	.0974	.1188
.35	.0002	.0009	.0036	.0191	.0565	.0895	.1053
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.60	.0002	.0009	.0032	.0148	.0359	.0455	.0412
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.70	.0002	.0008	.0029	.0124	.0268	.0299	.0234
.75	.0002	.0008	.0027	.0112	.0225	.0233	.0168
.80	.0002	.0007	.0025	.0099	.0185	.0176	.0117
.85	.0002	.0007	.0023	.0086	.0148	.0129	.0079
.90	.0002	.0006	.0021	.0074	.0116	.0092	.0051
.95	.0002	.0006	.0019	.0062	.0088	.0063	.0032
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1.10	.0001	.0004	.0012	.0033	.0033	.0017	.0007
1.15	.0001	.0004	.0010	.0025	.0022	.0010	.0004
1.20	.0001	.0003	.0008	.0019	.0014	.0006	.0002
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1.30	.0001	.0002	.0005	.0010	.0005	.0002	.0001
1.35	.0001	.0002	.0004	.0007	.0003	.0001	.0000
1.40	.0001	.0002	.0003	.0005	.0002	.0001	.0000
1.45	.0000	.0001	.0002	.0003	.0001	.0000	.0000
1.50	.0000	.0001	.0002	.0002	.0001	.0000	.0000
1.55	.0000	.0001	.0001	.0001	.0000	.0000	.0000
1.60	.0000	.0001	.0001	.0001	.0000	.0000	.0000
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1.70	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.75	.0000	.0000	.0000	.0000	.0000	.0000	.0000

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13. ABSTRACT

This report gives procedures for evaluating the operating characteristic curves and associated measures of dependent mixed (variables - attributes) acceptance sampling plans for the case of single specification limit, and known standard deviation, assuming a normal distribution. A useful generalized dependent plan is developed, using two attributes acceptance numbers rather than one. Also included is a comparison of dependent mixed plans with other types of acceptance sampling plans.

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	ROLE	WT	ROLE	WT	ROLE	WT
Mixed Sampling Plans Variables - Attributes Sampling Plans Sampling Inspection Plans Acceptance Sampling						

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